Inquiry Skills and Problem Solving Efficacy and Performance of Pre-Service Mathematics Teachers

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Abstract

Adequate acquisition and development of high-level cognitive skills is an integral component of teaching and learning. The study looks into inquiry skills in the context of mathematical investigation and problem solving efficacy and performance of 41 randomly selected pre-service mathematics teachers (PSTs) in two state universities in northern part of Panay Island using validated and pilot-tested researcher-made instruments. Statistical analyses were used to summarize the levels of performance and relate inquiry skills, problem solving efficacy and performance; and thematic analysis for difficulties and misconceptions of PSTs on mathematical inquiry and problem solving. The pre-service mathematics teachers have developing problem solving efficacy implying the need to reconcile their optimism in their ability to solve problems and their actual performance through relevant learning experience. As they are yet to attain the proficient level of skills in problem solving and mathematical inquiry, the development of their skills in these competencies may have been inadequate to effectively facilitate acquisition of learning, understanding and transfer of knowledge and skills among learners expected from future teachers. The ability of pre-service mathematics teachers to formulate mathematical concepts and principles operationally and prove these mathematical statements are not adequately developed as they have difficulties in exploring mathematical situations, employing appropriate reasoning in analyzing and abstracting patterns, and formulating generalizations. There are complementary associations between inquiry skills and problem solving performance such that the development of one implies the need to develop the other. Learning difficulties and misconceptions are attributed to inadequate development of fundamental skills necessary towards successful problem solving and mathematical investigation.

Keywords: Problem solving efficacy, pre-service teacher, inquiry skills, mathematical investigation

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Introduction

Education in the 21st century is confronted with challenges and demands of a fast-changing society necessitating the development of skills in all aspects of learning and daily life—learning and innovations, communication and collaboration, information and media technology, and life and career skills (Bilbao, Corpuz, Llagas, & Salandanan, 2012). An aspiring teacher needs to acquire not only the competence on what and how to teach but also has to reinvent himself beginning with a confidence and proper mindset as crucial aspect of becoming capable to answer the call of effective teaching.

The implementation of the K to 12 curriculum in the Philippine basic educational system likewise induces paradigm shifts in educational structure in all levels in the country. The new curriculum is designed to adhere to the standards and principles of being learner-centered, inclusive and developmentally appropriate, relevant, responsive and research-based, culture-sensitive, contextualized and global, and shall use pedagogical approaches that are constructivist, inquiry-based, reflective, collaborative, and integrative (R. A. No. 10533, 2013).

In a parallel stance, the Commission on Higher Education (CHED) requires outcomes-based education (OBE) framework in all institutions of higher learning. Learning, to be relevant, has to be authentic. That is, graduates are expected to possess the qualities that make them globally competitive in life, in their career and lifelong learning. For teacher education institutions (TEIs), the challenge rests on preparing would-be teachers in their prospective teaching careers. How well the teachers effect the desired educational innovations and outcomes is rooted from their pre-service education.

To facilitate the development of intended learning outcomes a set of skills and disposition have to be considered. Foremost of these are the inquiry and problem solving skills vis-ŕ-vis self-efficacy. Self-efficacy (efficacy expectations) refers to personal beliefs about one's capabilities to learn or perform actions at designated levels (Schunk 2012). It is a belief about what one is capable of doing; it is not the same as knowing what to do. In gauging self-efficacy, individuals assess their skills and their capabilities to translate those skills into actions. Self-efficacy is a key to promoting a sense of agency in people that they can influence their lives (Bandura, 1997, 2001). Self-efficacy and outcome expectations do not have the same meaning (Schunk & Zimmerman, 2006). Self-efficacy refers to perceptions of one's capabilities to produce actions; outcome expectations involve beliefs about the anticipated outcomes of those actions. Students may believe that a positive outcome will result from certain actions but also believe that they lack the competence to produce those actions.

There are many reasons for teacher educators to be focused on efficacy. Efficacy is linked to student achievement (Roof, 2015). Teachers with a high level of efficacy feel a sense of responsibility for student achievement (Hoy, 2000). Moreover, efficacy impacts motivation and performance (Tschannen-Moran, Hoy, & Hoy, 2008). For teachers themselves, the question on how a teacher's sense of efficacy affects his teaching is likewise a subject of several research investigations through the years. Jerald (2007) in his review of research identified some behaviors found to be related to a teacher's sense of efficacy. Teachers with stronger sense of efficacy tend to exhibit greater levels of planning and organization; are more open to new ideas and are more willing to experiment with new methods to better meet the needs of their students; are more persistent and resilient when things do not go smoothly; are less critical of students when they make errors; and are less inclined to refer a difficult student to special education (Protheroe, 2008).

For students, educators have long recognized that students' beliefs about their academic capabilities play an essential role in their motivation to achieve (Zimmerman, 2000). This supports Bandura's social cognitive theory where he discussed human motivation primarily in terms of outcome expectations.

Problem solving is the heart of learning mathematics. Through problem solving, the students are able to experience what mathematics is like, what mathematicians do in order to solve a problem or formulate a new mathematical concept. As students undergo the process of problem solving, a great deal of personal beliefs to their capabilities may be necessary. Self-efficacy beliefs provide students with a sense of agency to motivate their learning through use of self-regulatory processes as goal setting, self-monitoring, self-evaluation, and strategy use. As problem solving requires these cognitive and metacognitive tasks, a specific set of efficacy indicators may be determined, that is, problem solving efficacy. It has been noted that the more capable students judge themselves to be, the more challenging the goals they embrace (Zimmermann, Bandura, & Martinez-Pons, 1992); thus, problem solving efficacy plays pivotal role as students transition from one level of problem solving to another with increasing complexity.

It is from these perspectives that the study investigated the extent of pre-service teachers' inquiry skills, problem solving performance and problem solving efficacy. The possible relationship between these variables and the difficulties and misconceptions on inquiry and problem solving were closely looked into.

Inquiry skills are at the same level with decision making, investigation and problem solving skills making them one of the metacognitive skills on the basis of Bloom's cognitive domain. But what is inquiry? What is its role in education, particularly mathematics teaching and learning?

Inquiry, whether used in education or daily life, refers to the process of 'seeking knowledge by asking questions' (Artigue & Baptist, 2012). It is a philosophical approach to teaching and learning that is founded on constructivist research and methods as it engages learners in investigations leading to in-depth understanding that permeates across various disciplines (Saskatchewan Ministry of Education, 2011). Inquiry capitalizes on the young learners' inherent sense of curiosity and wonder, as it enables them to become active participants in the quest for meaning and understanding while drawing on the diverse backgrounds, experiences and interests.

Mathematical inquiry-based learning is often carried out through mathematical investigations and modelling activities. Mathematical investigations (MI) is a divergent process that begin with the exploration of a situation in order to generate mathematical principles (Palomo, 2014) along the way, employing critical and creative thinking skills. Mathematical investigation consists of the following stages: exploring a mathematical situation; identifying problems; forming conjectures; testing and verifying conjectures; justifying or proving conjectures; summarizing results; and making extension.

It is from these perspectives the study aimed to investigate: the extent of pre-service teachers' inquiry skills, problem solving performance and problem solving efficacy; relationships between these variables; and their difficulties and misconceptions on inquiry and problem solving. The results of the study provided the framework for the development of instructional materials on problem solving and mathematical investigations.

The study is anchored on the theory of constructivism in learning, Piaget's cognitive development, Bandura's social cognitive theory, Ohlsson's skill acquisition model, and Polya's heuristics on problem solving. Constructivism is an epistemology asserting that individuals form or construct much of what they learn and understand (Schunk, 2012). Rather than viewing knowledge as truth, constructivists construe it as a working hypothesis. Knowledge is not imposed from outside people but rather formed inside them. Hence, in this study, the exploration of inquiry skills and different approaches by the respondents in solving mathematical problems follow from this perspective. This theoretical perspective is in agreement with Piaget's cognitive development theory which states that learning is developmental relative to psychological development of a person. Bandura's social cognitive theory asserts that learning occurs in a social environment such that knowledge, rules, skills, strategies, beliefs, attitudes and values are learned primarily by modelling and observing others. One of the key concepts in this theory is that of triadic reciprocality. Bandura discussed human behaviour within this framework of reciprocal interactions among behaviors, environmental variables, and personal factors such as cognitions and efficacy. For instance, as students (person) work on tasks (environment), they note their progress (behaviour) toward their learning goals such as completing assignments, finishing sections of a term paper. Such progress indicators convey to students that they are capable of performing well and enhance their self-efficacy for continued learning (Schunk, 2012).

In this study, the problem solving and inquiry exercises provide the learning experience for the pre-service teachers to apply their learned knowledge and skills. Their interaction with the learning experience is likewise regulated by personal factors, which in this study, the problem solving efficacy. Polya's heuristics on problem solving, on the other hand, provides the template with which the respondents obtain the solution to the mathematical problems.

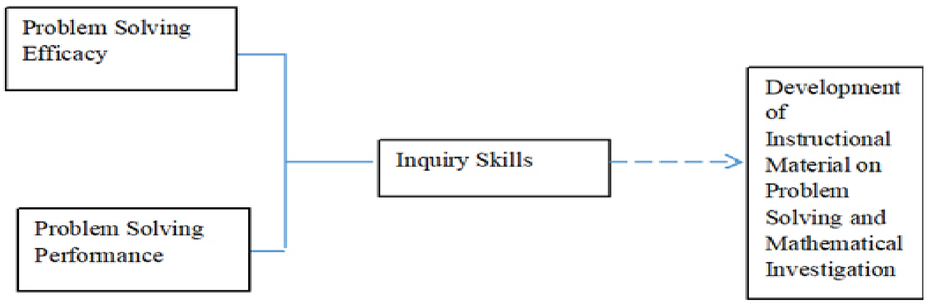

Figure 1. Pre-service teachers problem solving performance as related to problem solving efficacy, and inquiry skills as inputs to the development of problem solving and mathematical investigations module

Methodology

The study is of correlational research design to determine pre-existing causative relationships among the inquiry skills and problem solving efficacy and performance of pre-service mathematics teachers. Thematic analysis was also used to determine the difficulties and misconceptions of PSTs in performing these skills.

The participants were the 41 randomly selected pre-service mathematics teachers who have taken a course on problem solving and mathematical investigations and were officially enrolled during SY 2018-2019 in the two campuses of two state universities in the northern Panay.

The research instruments consisted of 16-item 9-point scale on problem solving efficacy ; 10-item problem solving skills test outlined after Polya's 4-stage problem solving approach: Understanding the Problem, Devising a Plan to Understand the Problem, Implementing a Solution Plan, and Reflecting on the Problem; and 8-item inquiry skills test covering the major stages of mathematical investigations: Exploring mathematical situation, Formulating conjecture, Verifying and testing conjecture, and Justifying or proving conjecture. The instruments were subjected to content validation and test-retest reliability procedures.

Analytic rubrics were used to score the performance of the respondents in problem solving and mathematical inquiry. Two of the researchers independently scored the responses of the participants. Mean, standard deviation and coefficient of correlation were used to quantitatively analyse the data. Inferential analyses were made at 0.05 level of significance. An interpretation scale based on mean score was used in interpreting problem solving efficacy and a scale based on mean percentage rating was used for inquiry and problem solving skills interpretation. Content and thematic analyses were used to identify the difficulties, gaps and misconceptions of the pre-service mathematics teachers in inquiry and problem solving.

Results and Discussion

Inquiry Skills of PSTs

The inquiry skills of pre-service mathematics teachers in terms of the stages of mathematical investigation as shown in Table 1 were generally at the beginning level (M = 11.85, SD = 4.95, PR = 42.75) indicating inadequate development of skills necessary to perform independently mathematical inquiry. Of the four key stages of mathematical investigation, only in verifying and testing conjectures that the PSTs obtained a developing level of performance (M = 4.56, SD = 1.58, PR = 57.00). Proving or justifying conjectures (M = 0.77, SD = 0.53, PR = 9.63) and formulating conjectures (M = 2.10, SD = 1.07, PR = 26.25) were shown to be the most difficult aspects of conducting mathematical inquiry. This prevailing inadequate development of these higher order mathematical skills were earlier noted as pre-service mathematics teachers were incapable to carry out formal proof that utilize sufficient and mathematically appropriate approach (Schwarz & Kaizer, 2008), experienced mathematics teachers were nearing mastery level in teaching proof (Pasigna & Herrera, 2014), and those who perceived mathematics as a set of fixed rules tend to have low level of knowledge in both content and pedagogy (Tatto, 2013).

Table 1. Inquiry skills of pre-service mathematics teachers.

INQUIRY SKILLS	TOTAL POINTS (n)	*M*	*SD*	PERCENT AGE RATING	VERBAL INTERPRETAT ION
Exploring Mathematical Situations	8	3.66	2.4 2	45.75	Beginning Level
Formulating Conjectures	8	2.10	1.0 7	26.25	Beginning Level
Testing/ Verifying Conjectures	8	4.56	1.5 8	57.00	Developing Level
Proving Conjectures	8	0.77	0.5 3	9.63	Beginning Level
TOTAL	**32**	**11.85**	**4.9 5**	**42.75**	**BEGINNING LEVEL**

Note: Interpretation is based on the following scale: 87.51 – 100.00 (Advanced), 75.01 – 87.50 (Proficient), 62.51 – 75.00 (Approaching Proficiency), 50.00 – 62.50 (Developing), and Below 50.00 (Beginning)

Problem Solving Efficacy of PSTs

The extent of problem solving efficacy of pre-service mathematics teachers was determined using a 9-point scale developed by the researchers in which qualitative descriptions range from nothing (1) to very little (2) to a great deal (9). The PSTs generally possessed a "quite a bit" or moderate extent of problem solving efficacy (M = 6.35, SD = 1.26) indicating a degree of confidence in problem solving that is on the process of development characterized by their optimism and potential to improve understanding of mathematical problems (M = 7.15, SD = 1.64), critical thinking ability (M = 6.85, SD = 1.44), motivate oneself in gaining interest in problem solving (M = 6.68, SD = 1.78), and engage oneself in problem solving task. The PSTs indicated less confidence in their ability to formulate plans in solving mathematical problems (M = 5.98, SD = 1.74), get through to the most difficult mathematical problems (M = 6.02, SD = 1.56), work well on non-routine problems (M = 6.07, SD = 1.47), verify the accuracy of results in the strategies used (M = 6.07, SD = 1.56). These results suggest that while the PSTs possessed a proper mindset directed towards successful problem solving, they acknowledged the limitations of their skills in performing actual and challenging problem solving.

Problem Solving Performance of PSTs

Investigating the problem solving performance of pre-service mathematics teachers employed Polya's 4-stage approach in solving both routine and non-routine mathematical problems. As shown in Table 2, the overall problem solving performance of PSTs was at the beginning level (M = 67.02, SD = 34.57, PR = 33.51) indicating heterogeneous abilities of the respondents and inadequate level of development of skills that require further assistance for transfer of learning and skills. In terms of the two categories of problems, the PSTs performance were also at the beginning level in both routine problems (M = 44.10, SD = 22.15, PR = 44.10) and non-routine problems (M = 22.93, SD = 15.17, PR = 22.93) with the latter suggesting difficulties among PSTs in getting through challenging mathematical problems that require the application of higher order thinking skills.

Table 2. Problem solving performance of pre-service mathematics teachers.

PROBLEM CATEGORIES	TOTAL POINTS (n)	*M*	*SD*	PERCENT AGE RATING	VERBAL INTERPRETAT ION
Routine Problems	100	44.10	22.1 5	44.10	Beginning Level
Non-routine Problems	100	22.93	15.1 7	26.25	Beginning Level
TOTAL	**200**	**67.02**	**34.5 7**	**33.51**	**BEGINNING LEVEL**

Note: Interpretation is based on the following scale: 87.51 – 100.00 (Advanced), 75.01 – 87.50 (Proficient), 62.51 – 75.00 (Approaching Proficiency), 50.00 – 62.50 (Developing), and Below 50.00 (Beginning)

In terms of Polya's heuristics, results presented in Table 3 showed that the PSTs seemed to focus on implementing a solution plan (M = 21.83, SD = 8.49, PR = 43.66) and devising a plan to solve a problem (M = 19.15, SD = 12.04, PR = 38.30) while they indicated minimal treatment on understanding the problem (M = 15.44, SD = 8.54, PR = 30.88) and reflecting on the nature of the problem and the results obtained (M = 10.61, SD = 11.56, PR = 21.22). It can be surmised that the PSTs may have confusion on problem solving and solving problem. Problem solving is an art of exploring, as creatively and critically, the approaches that may lead to solutions while solving problem is finding a solution to that satisfies what the problem requires.

Table 3. Problem solving performance of PSTs in terms of Polya's heuristics.

POLYA'S HEURISTICS	TOTAL POINTS (n)	*M*	*SD*	PERCENT AGE RATING	VERBAL INTERPRETAT ION
Understandin g the Problem	50	15.44	8.54	30.88	Beginning Level
Devising a Plan	50	19.15	12.0 4	38.30	Beginning Level
Implementing a Solution Plan	50	21.83	8.49	43.66	Developing Level
Reflecting on the Problem	50	10.61	11.5 6	21.22	Beginning Level
TOTAL	**200**	**67.02**	**34.5 7**	**33.51**	**BEGINNING LEVEL**

Note: Interpretation is based on the following scale: 87.51 – 100.00 (Advanced), 75.01 – 87.50 (Proficient), 62.51 – 75.00 (Approaching Proficiency), 50.00 – 62.50 (Developing), and Below 50.00 (Beginning)

Relationships among Problem Solving Efficacy and Performance, and Inquiry Skills

Problem solving efficacy and problem solving performance. There exists no significant relationship between problem solving efficacy and overall problem solving performance of pre-service mathematics teachers (-0.169, p = 0.29). This suggests that the PSTs' beliefs on their problem solving ability is not consistently associated with their actual competence in problem solving. No significant relationship was also noted between problem solving efficacy and the PSTs performance in both routine (-0.214, p = 0.180) and non-routine problems (-0.154, p = 0.336).

Problem solving efficacy and inquiry skills. The problem solving efficacy of pre-service mathematics teachers is not significantly related to their overall inquiry skills (0.231, p = 0.147). This indicates that the PSTs perception of their ability in problem solving is independent of their ability to perform mathematical investigations. There are two stages of mathematical inquiry, however, where problem solving efficacy are significantly associated with, formulating conjectures (0.395, p = 0.011) and justifying or proving conjectures skills (, p = 0.000). Here efficacy seems to have influence on the PSTs ability to construct an inference from exploration of a mathematical situation then justifying such inference.

Problem solving performance and inquiry skills. Problem solving and inquiry are both demonstrations of actual competence in higher order thinking skills of PSTs. Results show that there exists a moderately strong, positive and significant relationship between problem solving and inquiry skills (0.649, p = 0.000) which accounts for about 42.12 percent of the variance in one variable that is explained by the other. Inquiry skills are also associated to their abilities in solving routine (0.590, p = 0.000) and non-routine problems skills (0.668, p = 0.000). This implies that developing the problem solving skills produces complementary results in the mathematical inquiry skills of PSTs.

Difficulties and Misconceptions in Inquiry

In exploring mathematical situations, the results showed that pre-service teachers mostly employed spontaneous explorations when presented with mathematical situations. While this form of exploring mathematical situations may lead to the identification of possible problems to be pursued, it is the systematic exploration that is essential in investigating patterns and relationships in order to arrive at a particular generalization in an investigation.

In the second stage of mathematical inquiry the investigator formulates conjectures or tentative explanations based on observed patterns and relationships. It was found that the pre-service mathematics teachers possess insufficient skills in abstracting patterns to formulate appropriate conjectures, thus ensuing the inability to express the conjectures clearly with appropriate mathematical language.

In testing and verifying conjectures, the respondents employed less systematic approaches. In proving conjectures, the respondents have misconceptions in justifying a conjecture in terms of specific cases, and the inability to write adequate and coherent proofs. Figure 2 below shows examples of the respondents' misconception on verifying a formula using specific cases as an approach to proving.

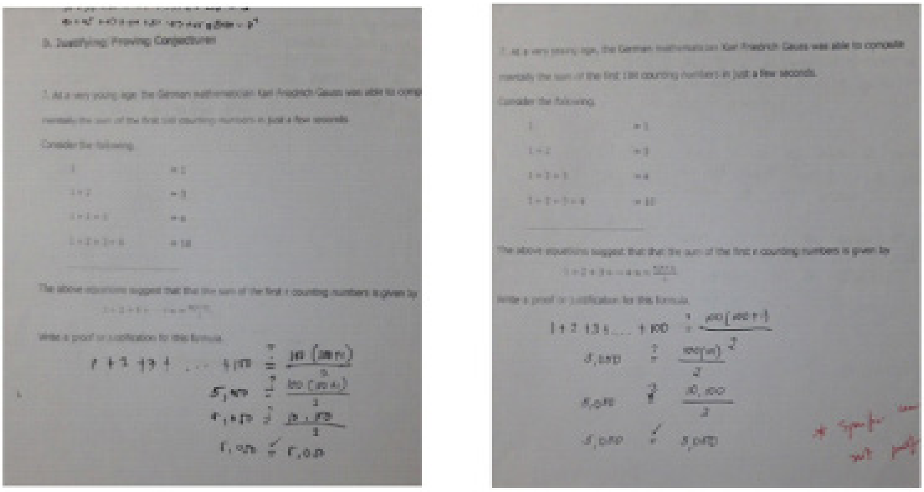

Figure 2. Attempts made by respondents to prove a statement by employing specific cases.

Difficulties and Misconceptions in Problem Solving

Using Polya's heuristics in problem solving, the beginning level of performance of the PSTs in problem solving may be attributed to their inability to pay attention to details in each of the stages as they have been exposed to a problem solving experience where getting the right solution matters. Shown in the following Figure 3 is a sample of such approach employed by the respondents.

In Understanding the problem stage, the PSTs indicated difficulty in paraphrasing the statement of the problem and indicated fewer attempts in determining whether there are irrelevant or missing information in the problem. Most responses in this stage are focused only on identifying the goal and the required information.

In Devising a plan to solve a problem, the PSTs indicated inadequate knowledge on how the chosen strategies work. The identified strategies were those that are commonly used: using equation, trial and error and guess and check. These reflect the PSTs seemingly lack of creativity in identifying several approaches that may be used in working out acceptable solutions.

In Implementing a solution plan, PSTs indicated less systematic approach in carrying out the identified strategies. The relationships implied in the problem are not translated into correct equations with quantities involved not properly represented.

In reflecting on the problem, the PSTs indicated very limited attempt in checking whether the obtained value satisfies the assumptions or required solution.

None of the respondents identified other approaches that may work in solving the problem and very few stated the actual solution to the problem.

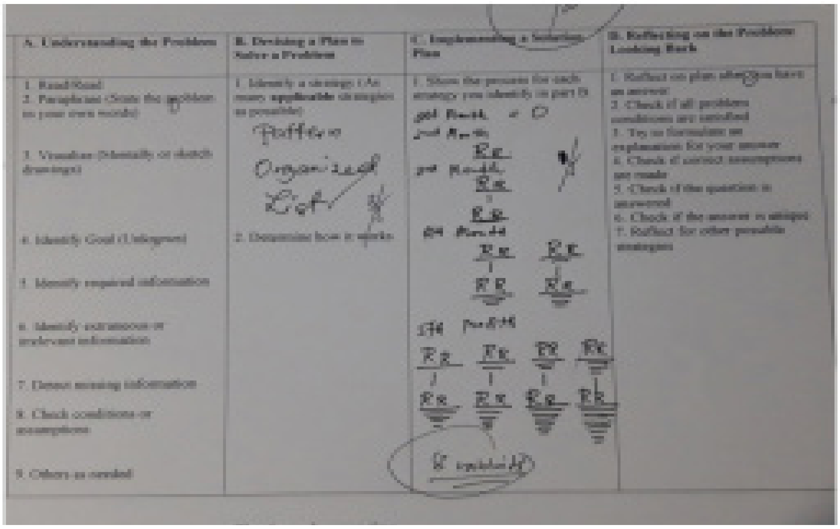

Figure 3. One of the respondents' approach in solving a problem. Despite the structured procedure in carrying out problem solving, the respondent proceeded to a haphazard presentation of the solution.

Conclusions

The pre-service mathematics teachers have developing problem solving efficacy. There is a need to reconcile their optimism in their ability to solve problems and their actual performance through relevant problem solving experience.

Pre-service mathematics teachers are yet to attain the proficient level of skills in problem solving and mathematical inquiry. The development of their skills in these competencies may have been inadequate to effectively facilitate acquisition, understanding and transfer of knowledge and skills among learners expected from future teachers.

The ability of pre-service mathematics teachers to formulate mathematical concepts and principles operationally and proving these mathematical statements are not adequately developed as they have difficulties in exploring mathematical situations, employing appropriate reasoning in analyzing and abstracting patterns, and formulating generalizations.

There are complementary associations between inquiry skills and problem solving performance such that the development of one implies the need to develop the other.

Learning difficulties and misconceptions are attributed to inadequate development of fundamental skills necessary towards successful problem solving and mathematical investigation.

Recommendations

Education policymakers, curriculum designers, and school administrative officials may introduce curricular innovations and programs intended for the development of inquiry, and problem solving skills, and other relevant skills of mathematics educators and future teachers in teacher education institutions.

Mathematics educators in teacher education institutions may consider the integrated approach in their teaching of mathematics courses to pre-service mathematics teachers. This may give emphasis on the development of inquiry and problem solving skills.

Textbook writers and instructional material developers may draw insights from the findings of the study in designing textbooks and other learning materials that promote inquiry and problem solving-based mathematics education.

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